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# Forward and Backward Intergenerational Goods: Why Is Social Security Good for the Environment?

By ANTONIO RANGEL\*

This paper studies the ability of nonmarket institutions to invest optimally in forward intergenerational goods (FIGs), such as education and the environment, when agents are selfish or exhibit paternalistic altruism. We show that backward intergenerational goods (BIGs), such as social security, play a crucial role in sustaining investment in FIGs: without them investment is inefficiently low, but with them optimal investment is possible. We also show that making the provision of BIGs mandatory crowds out the voluntary provision of FIGs, and that population aging can increase investment in FIGs. (JEL H0, H3, H4, H5, H6, D1, D7).

"Why should I care about future generations? What have they done for me?" (*Addison*)

"Be nice to your children, they will pick your nursing home." (*Anonymous bumper sticker*)

Every society uses a range of nonmarket institutions to decide how much to invest in future generations. A prominent example is the government and the decision of how much to invest on intergenerational (IG) public goods<sup>1</sup> such as

<sup>1</sup> See Todd Sandler and Kerry Smith (1976) and Sandler (1978, 1982) for an early development of the concept of "intergenerational goods," and Joaquim Silvestre (1994, 1995) for a more recent reference.

environmental preservation and pure science. These programs entail a transfer to future generations since they are financed with taxes on present generations and their benefits are longlived. Another prominent example is the family. Every generation of parents decides how much to invest in their children. Investments include the cost of (public and private) education, and the myriad of other sacrifices that parents make for their children.

These examples have a common structure. First, IG exchange takes place in an infinitely lived organization that has an overlapping generations (OLG) structure. Second, present generations have to decide how many resources to devote to investments that disproportionately benefit future generations. Third, once the investments are made, future generations cannot be excluded from the benefits that are generated.<sup>2</sup> Fourth, since future generations have not yet been born, present and future generations cannot negotiate binding contracts that reimburse present generations for the cost of the investment.<sup>3</sup> Fifth, membership in the

<sup>2</sup> For example, once the environmental public good has been provided, all the members of future generations benefit from it. Similarly, once children grow into adulthood, and are in a position to compensate the parents, the investments received in childhood are sunk and cannot be removed.

<sup>3</sup> In the case of the family, children are alive at the time of the parent's investment. However, they have not been "born" as economic agents since they cannot sign binding contracts.

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organization is not for sale: agents are born into the organization. As the following examples show, the last three characteristics are central to the problem because they rule out market-based solutions.

To see the role of excludability, consider the case of publicly traded companies. Every period present generations of stockholders decide how much to invest to increase future profits. Here, the benefits generated by IG investment are excludable since the share of profits received by an agent is proportional to the amount of stock that he owns. This organization satisfies characteristics (4) and (5), but not (3).<sup>4</sup> The presence of a stock market induces members to internalize the effect of investments on future profits since they raise the price at which the stock can be sold. As a result, this market institution typically generates optimal IG investment.

To see the role of exogenous membership, consider the case of a country club, which is an organization with an overlapping generations structure in which membership has to be purchased. This organization satisfies all of the characteristics listed above except exogenous membership. In contrast to the case of the firm, members cannot be excluded from investments such as golf courses. We can think of these organizations as IG clubs.<sup>5</sup> In the absence of externalities across clubs, a market solution in which club membership is a tradeable asset could generate optimal levels of investment.

Finally, to see the role of incomplete IG contracting, consider the case of the family. Even with selfish generations, an efficient amount of investment would take place if children and parents could sign binding contracts.<sup>6</sup> In the absence of "transaction costs," an IG form of the Coase Theorem would arise: children would arise:

dren would commit to compensate their parents for the cost of the optimal investment, and parents would have an incentive to provide these investments. The problem is that in many organizations such contracts are not possible. In the case of the family, the legal system precludes these types of contracts since children lack the independence and understanding required to evaluate them.

This paper develops a stylized model of IG exchange to study the conditions under which nonmarket institutions are able to generate Pareto-optimal levels of investment. Agents live for three periods: young, middle-aged, and old. Every period the middle-aged agent decides how much to invest in a forward intergenerational good (FIG) that benefits future generations, but not himself. Although the role of altruism is studied in the paper, it is useful to start with the case of selfish generations.

We start with an immediate observation. If the only decision made every period is how much to invest in FIGs, no investment takes place. The intuition is straightforward. Agents benefit from investments in FIGs made by past generations, but not by investments made after they are born. Thus, they have no incentive to invest in FIGs, and no FIGs are produced. We can conclude that optimal IG investment cannot arise when the only decision made by the organization is how much to invest in future generations.

Fortunately, in addition to choosing how much to invest in future generations, most nonmarket institutions also make decisions about backward IG exchange. For example, the government transfers resources to the elderly through the social security system, and families take care of their elderly parents. The main insight of this paper is that the presence of backward IG goods plays a crucial role in sustaining investment in future generations: without backward exchange, investment in FIGs is inefficiently low; but with it, even optimal investment by selfish generations is possible. The crucial insight comes from the literature on multimarket contact in industrial organization [see Jonathan Bendor and Dilip Mookherjee (1990) and B. Douglas Bernheim and Michael D. Whinston (1990)], which has shown that linkages across games play an important role in sustaining cooperation.

<sup>&</sup>lt;sup>4</sup> For the purpose of the example, we can think of membership as exogenous: everyone is a "member" of the organization but only agents with a positive amount of stock have a claim on the profits.

<sup>&</sup>lt;sup>5</sup> See Sandler (1982).

<sup>&</sup>lt;sup>6</sup> The "intergenerational Coasian theorem" described here requires present and future generations to be able to bargain, face to face, and sign binding contracts. The ability to bind future generations to transfer resources to present generations is not sufficient: if present generations do not have an incentive to invest in future generations without compensation, they also have an incentive to impose transfers on future generations without investing in them.

To study the relationship between forward and backward IG exchange, we analyze a stylized model in which, every period, the middleaged agent makes two decisions: (1) how much to invest in a FIG, and (2) how much to buy of a backward intergenerational good (BIG) that only benefits the elderly. Using this framework, in Section III we show that a link between BIGs and FIGs is essential for sustaining optimal levels of investment in future generations. We also show that the need for a link between BIGs and FIGs has the following implications. First, the social rate of return, risk characteristics, and horizon of the FIGs do not affect whether or not they are financed. Second, within some limits, population aging can increase public investment in FIGs. Finally, making the provision of BIGs mandatory can crowd out investment in future generations.

In Section V we explore the implications of the analysis for two important IG organizations, the government and the family. There we answer the question posed in the title: why is social security good for the environment? Here is the bottom line. If a majority of the electorate receives positive benefits from keeping the social security system, there are voting equilibria in which even selfish generations vote to invest in FIGs. In these equilibria, investment in future generations is supported by a link between BIGs and FIGs: present voters correctly believe that future voters' support of social security depends on whether or not they invest in FIGs.

#### I. Relation with the Literature

Using the BIGs and FIGs framework, the literature on nonmarket IG organizations can be divided into three strands.

The first strand studies organizations in which there is only *intra*generational exchange. Jacques Cremer (1986), David Salant (1991), Michihiro Kandori (1992), Lones Smith (1992), and Kenneth Shepsle (1999) study OLG organizations in which every period all of the agents simultaneously take an action that affects every one alive at the time, but has no effect on future generations. For example, in Cremer (1986), agents simultaneously choose how much effort to exert in production, and total output depends on the sum of the efforts. The main insight from this literature is that the standard "Folk Theorem" results extend to the OLG context: cooperation can be sustained as long as agents are patient and/or live long enough. The last condition is needed because agents in the last period of life cannot be given an incentive to cooperate. By contrast, the results developed in this paper are not limit results.<sup>7</sup>

The second strand of the literature studies organizations in which the exchange problem looks like a BIG. Here the organization chooses how much to produce of a good that is basically a transfer from the younger to the older generations. Consider, for example, the "Pension Game" in Hammond (1975),<sup>8</sup> in which he studies a standard OLG economy with two-period lifetimes. Agents have an endowment when young, but not when old, and have no access to a savings technology. Hammond shows that there are equilibria, similar to the one developed in Section III, subsection B, that sustain Paretoimproving transfers from young to old in every period. To study the political economy of payas-you-go social security, Hansson and Stuart (1989), Henning Bohn (1998), and Cooley and Soares (1999) extend this model to a setting in which agents live for more than two periods and decisions are made by majority rule. The insights from all of these papers are similar to the results for BIGs in Sections III and V.

Two important papers in this literature are Kotlikoff et al. (1988) and David Kreps (1990), who show that the presence of a sustainable BIG can be used to solve inefficiencies in the economy. Kotlikoff et al. (1988) study a standard OLG economy with two-period lifetimes in which every generation elects its own separate government. Each generational government faces a standard commitment problem: it would like to choose low capital tax rates but cannot credibly commit to do so. They show that the commitment problem can be overcome through the introduction of a self-sustainable "IG compact" in which every generation agrees to transfer a large sum to the previous generation as long as it has followed the compact and has chosen low capital tax rates for itself. Kreps

<sup>&</sup>lt;sup>7</sup> This is also true in Peter Hammond (1975), Laurence Kotlikoff et al. (1988), Ingemar Hansson and Charles Stuart (1989), and Thomas Cooley and Jorge Soares (1999), which are described below.

<sup>&</sup>lt;sup>8</sup> See also Narayana Kocherlakota (1998).

JUNE 2003

(1990) shows that the transfers can be used to overcome moral hazard problems. As we do here, these papers show that linking the provision of a BIG with something else can be beneficial; in our case to provide investment in future generations, in Kotlikoff et al. and Kreps to solve an *intra*generational incentive problem.

Next, several papers have studied organizations in which there are only FIGs and shown that underinvestment must take place. For example, Jacobus Doeleman and Sandler (1998) study investment in IG public goods in a finite OLG model and conclude that, with selfish generations, underinvestment takes place [see also Kotlikoff and Robert Rosenthal (1993) and David Collard (2000)]. By contrast, in this paper we study organizations in which both BIGs and FIGs are provided.

Finally, two other papers have argued that there is a link between forward and backward IG exchange. Gary Becker and Kevin Murphy (1988) suggest that it is possible to think of old age social insurance and education as a trade among generations: children receive an education from their parents and in exchange pay for their retirement benefits. However, their's is mostly an accounting argument. They do not study the sustainability of these arrangements, which is the focus of this paper. This is problematic because when children grow up they can default on their obligations. Michele Boldrin and Ana Montes (1998) have independently developed an analysis that is closely related.<sup>9</sup> They study the majority rule politics of pay-asyou-go social security and public education using an overlapping generations economy. Although there are some differences in the details of the model, both papers arrive at similar insights. In particular, their main result is analogous to Proposition 3 in this paper.

#### II. Model

Consider a simple model of an infinitely lived organization with an overlapping generations demographic structure. Each period t a new member, called generation t, enters the organization and stays there for three periods: t, t + 1, and t + 2. We say that the agent is young in

the first period, middle-aged in the second, and old in the third. Time is indexed by t = 1, 2, .... At time 1 there is also an old generation -1 that stays in the organization only for that period, and a middle-aged generation 0 that belongs to the organization in periods 1 and 2.

Every period t, the middle-aged generation t - 1 has to make two decisions: (1) how much to invest on a FIG that only benefits generation t + k, where  $k \ge 0$ ; and (2) how much to spend on a BIG that only benefits the current old.  $k \ge 0$  denotes the lag between the time the FIG is produced and the time the benefits are received. Let  $f_t$  denote the investment in FIGs in period t, and  $b_t$  the amount spent on BIGs. These costs are paid by generation t - 1.

Every generation receives endowments  $w^{\nu}$ ,  $w^{m}$ , and  $w^{o}$  of a private consumption good in each of the three stages of their lives. Agents can borrow and lend at the economy's interest rate *r*. The preferences of generation *t* are given by

(1) 
$$U(c^{y}, c^{m}, c^{o}) + F(f_{t-k}) + B(b_{t+2}),$$

where  $c^{y}$ ,  $c^{m}$ ,  $c^{o}$  denote their consumption when young, middle-aged, and old. In this specification of the model, the young of generation t + k are the only agents who benefit from the FIG produced at time t, and there is no IG altruism. We start with this stark case for expositional purposes. More complex FIGs and IG altruism are introduced later on. All the functions are twice continuously differentiable and increasing, U satisfies the usual strict concavity and Inada conditions, and F and B are strictly concave.

The following notation greatly simplifies the analysis. Let

(2) 
$$V(x) \equiv \underset{c^{y}, c^{m}, c^{o}}{\arg \max} U(c^{y}, c^{m}, c^{o})$$
  
s.t.  $c^{y}(1+r) + c^{m} + \frac{c^{o}}{(1+r)} = x$ ,

which denotes the indirect utility from the consumption of private goods for an agent who spends an amount of wealth *x*, where wealth is measured at middle-age.

The actions that are taken in the organization define an infinite game with overlapping generations of players. Every period t, generation t - 1 chooses

<sup>&</sup>lt;sup>9</sup> This paper is a revised version of Rangel (1997).

VOL. 93 NO. 3

(3) 
$$(b_t, f_t) \in \{(b, f) | 0 \le b + f \le \bar{w}, b \ge 0, \text{ and } f \ge 0\},\$$

where  $\bar{w} \equiv w^{y}(1 + r) + w^{m} + \frac{w^{o}}{(1 + r)}$  denotes the lifetime wealth of every generation (measured at middle-age). Let  $h_{t} = ((b_{1}, f_{1}), ..., (b_{t-1}, f_{t-1}))$  denote the history of actions taken up to period t. A strategy for generation t - 1, which takes an action in period t, is a function  $s_{t}(h_{t}) = (s_{t}^{B}(h_{t}), s_{t}^{F}(h_{t}))$  that specifies the amount of BIGs and FIGs purchased at any possible history. Let  $s = (s_{1}, s_{2}, ...)$  denote a profile of strategies for every generation. The payoff for generation t - 1, conditional on history  $h_{t}$ , is given by:

(4) 
$$\Pi_{t}(s|h_{t}) = V(\bar{w} - s_{t}^{B}(h_{t}) - s_{t}^{F}(h_{t})) + F(f_{t-(k+1)}) + B(s_{t+1}^{B}(h_{t}, s_{t}(h_{t}))).$$

Note that the payoff of generation t - 1 is affected only by a small number of the decisions taken in the organization:  $f_{t-(k+1)}$ ,  $b_i$ ,  $f_i$ , and  $b_{t+1}$ . We use subgame-perfect equilibrium as the solution concept.

#### **III. Results**

#### A. A Useful Tool

We start the analysis by deriving a useful result. Given any path  $\gamma = \{(\tilde{b}_t, \hat{f}_t)\}_{t=1}^{\infty}$  of BIGs and FIGs, define the following profile of simple trigger strategies (STSs):

(5) 
$$s_t(h_t) = \begin{cases} (\hat{b}_t, \hat{f}_t) & \text{if } \mu(h_t) = C \\ (0, \hat{f}_t) & \text{if } \mu(h_t) = P. \end{cases}$$

 $\mu$  is defined recursively as follows:  $\mu(h_1) = C$ and

(6) 
$$\mu(h_t) = \begin{cases} C & \text{if } \mu(h_{t-1}) = C \text{ and} \\ (b_{t-1}, f_{t-1}) = (\hat{b}_{t-1}, \hat{f}_{t-1}) \\ C & \text{if } \mu(h_{t-1}) = P \text{ and} \\ (b_{t-1}, f_{t-1}) = (0, \hat{f}_{t-1}) \\ P & \text{otherwise.} \end{cases}$$

The idea behind STSs is straightforward.  $\mu$  is a flag that keeps track of whether the organization is in a cooperative or in a punishment phase. In the cooperative phase agents produced the level of BIGs and FIGs prescribed by  $\gamma$ . In a punishment phase they only produce the prescribed level of FIGs. Note that if every generation plays the STS then  $\gamma$  is the outcome of the game.

PROPOSITION 1: A path  $\gamma$  of BIGs and FIGs can be sustained as a subgame-perfect equilibrium if and only if it can be sustained as a subgame-perfect equilibrium using STSs.<sup>10</sup>

This result is useful because it shows that to check if a particular path of BIGs and FIGs can be sustained, it is enough to test if it can be sustained using STSs. Note that STSs are not the only strategies that can be used. For example, if  $\gamma$  can be sustained using STSs, then it can also be sustained using "grim strategies" in which failure to produce the prescribed level of BIGs or FIGs ends cooperation forever. We focus on STSs because they have two appealing properties. First, they are simple. Second, the punishment phase lasts for only one period, and only the generation that failed to produce the prescribed level of BIGs and FIGs is punished.<sup>11</sup>

Using this tool it is easy to characterize the set of paths that can be sustained as a subgameperfect equilibrium.

PROPOSITION 2: A path  $\gamma$  of BIGs and FIGs can be sustained as a subgame-perfect equilibrium if and only if

(7) 
$$V(\bar{w} - \hat{b}_t - \hat{f}_t) + B(\hat{b}_{t+1})$$
$$\geq V(\bar{w}) + B(0) \text{ for all } t.$$

<sup>10</sup> All of the proofs are in the Appendix.

<sup>11</sup> Venkataraman Bhaskar (1998) shows that, in this type of overlapping generations game, the existence of cooperative equilibria depends crucially on the observability of the entire history of play. In particular, no cooperation is the unique equilibrium in pure strategies when generations can observe *at most* the actions of the last *n* predecessors.

# B. Characterization of Equilibria

We start with an immediate observation: no FIGs are produced in an organization in which no other decisions are made.<sup>12</sup> Thus, the presence of BIGs, or other forms of exchange that will be discussed below, are essential to generate positive investments in FIGs. This section explores in detail the relationship between BIGs and FIGs.

It is useful to start with an organization in which there are only BIGs. Consider for a moment a version of the model in which the only action chosen every period t is how much to spend on BIGs ( $f_t = 0$  for all t). Everything else remains unchanged. This generates a game in which  $\{b|0 \le b \le \bar{w}\}$  is the action set for each generation.

Consider any path  $\beta = {\{\hat{b}_t\}}_{t=1}^{\infty}$  of BIGs, and define a continuation surplus function given by

(8) 
$$S_t^B(\beta) \equiv [V(\bar{w} - \hat{b}_t) + B(\hat{b}_{t+1})] - [V(\bar{w}) + B(0)].$$

This measures the surplus generated by the IG trade implicit in  $\beta$ : the first term measures the payoff for generation t - 1 of producing  $\hat{b}_t$  and receiving  $\hat{b}_{t+1}$ , the second term measures its payoff at generational autarky where no BIGs are produced.

Note that the continuation surplus function is positive for all t if and only if the path  $\beta$ satisfies condition (7). Thus, we conclude that a path of BIGs  $\beta$  can be sustained as a subgameperfect equilibrium if and only if  $S_t^B(\beta) \ge 0$  for all t. This characterization is very intuitive. Every generation needs to decide how many BIGs to give to the old, and in exchange it receives some BIGs from the next generation. In a STS, a generation that does not provide the prescribed amount of BIGs for the previous generation is punished by not receiving any BIGs when it becomes old. Thus, the cost of not cooperating is equivalent to returning to generational autarky. This characterization says that a path  $\beta$  can be sustained only if the amount of BIGs received in old age outweighs, for every

<sup>12</sup> This can be seen by setting  $B(\cdot) = 0$  in condition (7), which implies that the only path that satisfies the inequality is  $\gamma = \{(0, 0)\}_{t=1}^{\infty}$ .

generation, the cost of financing the BIGs for the previous generation.

Now consider the level of BIGs that can be sustained as a stationary equilibrium. By Proposition 2, a stationary level of BIGs b can be sustained if and only if

(9) 
$$S_{st}^{B}(b) \equiv [V(\bar{w} - b) + B(b)] - [V(\bar{w}) + B(0)] \ge 0.$$

If  $B'(0) > V'(\bar{w})$ ,  $S_{st}^B(\cdot)$  takes positive values in the interval  $[0, b_B^{max}]$ , where  $b_B^{max}$  denotes the maximum level of BIGs that can be sustained. Furthermore,  $b_B^{max} > b_B^*$ , where  $b_B^*$  denotes the optimal stationary level of BIGs in the model in which there are no FIGs.<sup>13</sup> This has two implications that will be useful below. First, for any level of BIGs  $b \in (0, b_B^{max})$ , the stationary path generates a positive surplus: every generation is better off producing and receiving this level of BIGs than in generational autarky. Second, inefficient overproduction and underproduction of BIGs is possible.

Using these insights we now characterize the level of FIGs that can be sustained in the full model. For any path  $\gamma$  define the following continuation surplus function:

(10) 
$$S_t(\gamma) \equiv [V(\bar{w} - \hat{b}_t - \hat{f}_t) + B(\hat{b}_{t+1})]$$
  
  $- [V(\bar{w}) + B(0)]$   
  $= S_t^B(\gamma) - [V(\bar{w} - \hat{b}_t)$   
  $- V(\bar{w} - \hat{b}_t - \hat{f}_t)].$ 

This function measures the value of the exchange implicit in  $\gamma$  over generational autarky. It is equal to the continuation surplus of the exchange in BIGs implicit in  $\gamma$ , minus the utility cost of financing an amount of FIGs  $\hat{f}_t$ . Thus,  $S_t(\gamma) < S_t^B(\gamma)$  whenever  $\hat{f}_t > 0$ . Note that the continuation surplus does not measure "social surplus" since it excludes the benefits generated by FIGs.

$${}^{13}b_B^* \equiv \arg \max_b V(\bar{w} - b) + B(b).$$

#### VOL. 93 NO. 3

# **PROPOSITION 3:**

- (1) A path  $\gamma$  of BIGs and FIGs can be sustained as an equilibrium if and only if  $S_t(\gamma) \ge 0$  for all t.
- (2) A positive level of FIGs can be sustained in every period if and only if there exists a path of BIGs  $\beta$  such that  $S_t^{\text{B}}(\beta) > 0$  for all t.
- (3) If B'(0) > V'(w), then there are equilibria in which positive levels of FIGs are produced in every period.

Proposition 3 provides a simple but important insight. Positive investments in FIGs can take place even when present generations are selfish. However, three conditions are necessary. First, the members of the organization must also face another exchange problem that requires cooperation. BIGs are one such possibility, but as we will see in Section III, subsection E, not the only one. Second, the non-FIG dimension must be capable of generating cooperative trades that generate a positive continuation surplus. Third, the generations must play strategies that link cooperation in the non-FIG dimension with sufficient investment in future generations. With this link, an agent receives a BIG in old age only if he purchases the right level of BIGs for the previous generation and invests enough on FIGs.

Now consider the levels (b, f) of BIGs and FIGs that can be sustained as stationary equilibria. By Proposition 3, (b, f) can be sustained if and only if

(11) 
$$S_{st}(b, f) = [V(\bar{w} - b - f) + B(b)] - [V(\bar{w}) + B(0)] \ge 0.$$

Let  $f_{\text{max}}(b)$  denote the maximum amount of FIGs that can be sustained with a level of BIGs equal to *b*. This function is defined implicitly by the equation  $S_{st}(b, f) = 0$ . By the Implicit Function Theorem, the function is defined for any  $b \ge 0$ , is continuously differentiable, and its derivative is given by

(12) 
$$f'_{\max}(b) = \frac{B'(b) - V'(\bar{w} - b - f_{\max}(b))}{V'(\bar{w} - b - f_{\max}(b))}$$

Figure 1 characterizes the set of stationary equilibrium outcomes. The bell-shaped curve depicts the function  $f_{max}(b)$  for an organization

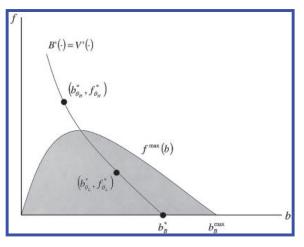


FIGURE 1. SET OF STATIONARY EQUILIBRIUM OUTCOMES

in which  $B'(0) > V'(\bar{w})$ . [If this is not the case,  $f_{\max}(b) \le 0$  for all b]. The figure also plots the locus  $B'(b) = V'(\bar{w} - b - f)$ , which is downward sloping and intersects the horizontal axis at  $b_B^*$ . The function  $f_{\max}$  increases until it intersects the locus, and then decreases until  $b_B^{\max}$ , which denotes the maximum level of BIGs that can be sustained. Since  $f < f_{\max}(b)$ implies that S(b, f) > 0, the area under the curve provides a complete characterization of the set of stationary equilibrium outcomes.

Note that the benefits generated by FIGs play no role in the characterization of this set. Changes in  $F(\cdot)$  affect the optimal level of FIGs, but not the amount of FIGs that can be sustained. This is somewhat counterintuitive. One would expect IG investments with a more advantageous benefit-cost ratio to be more likely to be financed; but this is not the case. As a result, investments in future generations that are relatively inexpensive are more likely to be funded than programs that generate larger net social benefits but are also more costly.

Can the stationary Pareto-optimal level of investment in FIGs be sustained? To answer this question it is useful to introduce a parameter  $\theta$ that affects the benefits generated by FIGs. Let

(13) 
$$(b_{\theta}^*, f_{\theta}^*) \equiv \underset{b,f}{\arg \max} V(\bar{w} - b - f)$$
  
+  $B(b) + \theta F(f)$ 

denote the optimal stationary level of BIGs and FIGs. As long as B and F satisfy the Inada

condition, the solution is interior and satisfies the FOCs

(14) 
$$B'(b^*_{\theta}) = V'(\bar{w} - b^*_{\theta} - f^*_{\theta}) = \theta F'(f^*_{\theta}).$$

Furthermore, we have that

(15) 
$$\frac{\partial f_{\theta}^{*}}{\partial \theta} > 0, \frac{\partial b_{\theta}^{*}}{\partial \theta} < 0,$$
  
and  $(b_{0}^{*}, f_{0}^{*}) = (b_{B}^{*}, 0).$ 

The locus  $(b_{\theta}^*, f_{\theta}^*)$  is depicted in Figure 1.<sup>14</sup> For small  $\theta$ ,  $(b_{\theta}^*, f_{\theta}^*)$  lies in the interior of the sustainable set and thus optimal production, and even inefficient overproduction, are possible. As  $\theta$  increases, the optimal level of FIGs  $f_{\theta}^*$  eventually increases beyond what can be sustained with the BIGs available in the organization. This establishes the following result.

PROPOSITION 4: Inefficient overproduction of FIGs can be sustained as a stationary equilibrium if and only if  $S(b^*, f^*) > 0$ . In this case, any level of FIGs  $f \in [0, \max_b f_{\max}(b)]$ , including the optimal stationary level  $f^*$ , can be sustained as a stationary equilibrium.

As Figure 1 starkly illustrates, the model generates a large number of equilibria: in some of them IG cooperation takes place, in others it does not. This is common in models of longlived institutions that use noncooperative game theory. The equilibrium set could be reduced by imposing equilibrium refinements like Markovian equilibrium or renegotiation proofness. However, given that so far game theory has not provided a fully satisfactory justification for such refinements, we proceed by characterizing the entire equilibrium set. Future theoretical developments might be able to identify variables that affect the coordination of expectations across generations, and thus rule out some of the equilibria.

We conclude this section with a comparison of forward and backward IG exchange. From a technological point of view, BIGs and FIGs are not that different. Both types of exchange require agents to provide a good that is valuable for another generation, and in exchange benefit from a good that is provided for them. However, from an incentive point of view, they are significantly different. First, even when BIGs and FIGs generate identical benefits [i.e., when  $B(\cdot) = F(\cdot)$ ], BIGs can be sustained in organizations that only make this type of decisions, but FIGs cannot be sustained in isolation. A positive level of investment in FIGs can arise only by linking them with BIGs. Second, the cost and benefits of BIGs are crucial in determining the level of BIGs and FIGs that can be sustained. By contrast, the benefits generated by FIGs play no role. Third, the optimal stationary level of BIGs is always sustainable. This is not the case for FIGs.

# C. What Type of FIGs Can Be Sustained?

The basic model ignores some important properties of FIGs. For example, environmental programs often generate benefits for multiple generations, including those making the investment, and have uncertain returns. In this section we show that these issues do not alter the insights obtained above.

Consider first the case of multiple beneficiaries and risk. For concreteness, consider a FIG for which the preferences of generation t are given by

(16) 
$$U(c^{y}, c^{m}, c^{o}) + F(\omega_{t}, \{f_{t-k}\}_{k=0}^{t-1}) + B(b_{t+2}),$$

where  $\omega_t$  is a random shock realized at the beginning of period *t*. In this case, the FIG produced at time *t* can benefit every future generation, including generation *t*.

It is straightforward to see that the path of sustainable FIGs has not changed. Given that FIGs are nonexcludable, past investments in FIGs do not affect the incentive constraint of the decision maker. As a result, the characterization provided in Propositions 3 and 4 still holds, and the set of stationary equilibrium outcomes is still the one depicted in Figure 1. The lesson is clear: risks, lags, and multiple beneficiaries affect the optimal level of investment in FIGs, but not the level that can be sustained.

<sup>&</sup>lt;sup>14</sup> It is given by the line  $B'(\cdot) = V'(\cdot)$ .

Now consider the case in which the generation investing in FIGs also benefits from them. For concreteness, suppose that the preferences of generation t - 1, the one making decisions in period t, are given by

(17) 
$$U(c^{y}, c^{m}, c^{o}) + F(f_{t-(k+1)}) + G(f_{t}) + B(b_{t+1}),$$

where G is concave, continuously differentiable, and increasing. The only difference with the basic model is that generation t - 1 now gets a direct benefit  $G(f_t)$  from investing in FIGs.

To analyze this case define a new continuation surplus function given by

(18) 
$$S_t^G(\gamma) \equiv [V(\bar{w} - b_t - f_t) + B(\hat{b}_{t+1}) + G(\hat{f}_t)] - [\max_f V(\bar{w} - f) + B(0) + G(f)]$$

The key difference with the previous case is that now generations want to produce some of the FIG even if cooperation breaks down. This is reflected in the second term, which is the payoff at generational autarky.

PROPOSITION 5: Consider the case in which FIGs generate benefits for the generation making the investment:

- (1) A path  $\gamma$  of BIGs and FIGs can be sustained as an equilibrium if and only if  $S_t^G(\gamma) \ge 0$  for all t.
- (2) If  $G'(0) > V'(\bar{w} b_B^{\max})$ , any stationary equilibrium that does not link BIGs and FIGs involves inefficient underproduction of FIGs.

Proposition 5 shows that underinvestment in FIGs still takes place unless two conditions are met. First, the agents must play strategies that link BIGs and FIGs. Second, the BIGs must generate a large enough continuation surplus that can be used to give incentives to invest beyond the short-sighted level.

A characterization of the set of stationary equilibrium outcomes illustrates this point. Figure 2 depicts the set for an environment in

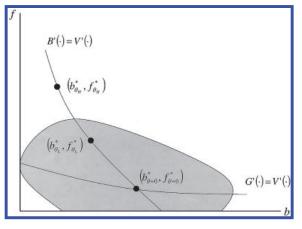


FIGURE 2. SET OF STATIONARY EQUILIBRIUM OUTCOMES WHEN FIGS BENEFIT PRESENT GENERATIONS

which  $G'(0) > V'(\bar{w})$  and  $B'(0) > V'(\bar{w} - f^{au})$ . The first condition guarantees that  $f^{au}$ , the amount of FIGs produced in generational autarky, is positive. The second condition guarantees that the maximum amount of FIGs that can be sustained is larger than  $f^{au}$ . Since the characterization is similar to the one for Figure 1 the details are omitted.

Note a few properties of the equilibrium set. First, (0, 0) is no longer an equilibrium. This follows from the fact that agents invest in FIGs in the absence of cooperation. Second, consider a parameterization of the preferences given by

(19) 
$$U(c^{y}, c^{m}, c^{o}) + \theta F(f_{t-(k+1)}) + G(f_{t}) + B(b_{t+1}),$$

where  $\theta$  measures the relative fraction of the benefits that are internalized by the decision maker. Let  $(b_{\theta}^*, f_{\theta}^*)$  denote the optimal stationary level of production. As before, the optimal level of FIGs is sustainable when  $\theta$  is small but not when it is large. Finally, the equilibrium set cuts the horizontal axis (i.e., there is an equilibrium in which no FIGs are produced) if and only if

(20) 
$$\max_{b} V(\bar{w} - b) + B(b) + G(0)$$
$$\geq \max_{f} V(\bar{w} - f) + B(0) + G(f).$$

Thus, when the surplus generated by BIGs is large relative to the value of FIGs, there are pathological equilibria in which no generation invests in FIGs even though they benefit directly from those investments.

# D. The Role of Altruism

Another concern with the basic model is the assumption of selfish generations. The effect of altruism on the analysis depends on the specific form that it takes. In particular, one needs to distinguish between paternalistic and nonpaternalistic altruism. With paternalistic altruism, the level of FIGs that future generations consume enters as an argument in the utility function of present generations. With nonpaternalistic altruism, it is the utility level of future generations that is internalized by present generations.

The analysis of paternalistic altruism is equivalent to the case discussed at the end of the previous section. The only difference is that the function G is now interpreted as altruism, instead of a direct benefit from consuming the FIG. Therefore, in this case a link between BIGs and FIGs is still needed to generate optimal levels of production.

The case of nonpaternalistic altruism is qualitatively different. Consider, in particular, the dynastic model in Robert Barro (1974). In this model there is no IG exchange problem. The organization behaves like an infinitely lived agent that perfectly internalizes the future spillovers of investing in FIGs. Thus, in applications for which the dynastic model is a good description of behavioral motives, the issues studied in this paper do not arise.

Casual evidence suggests that IG altruism is at work in most of the applications of this model: voters care about the future of humanity and parents care about their children. However, the key question is which is the form that their altruism takes. Although a lot of work remains to be do done in this area, and a consensus does not exist yet, some existing evidence suggests paternalistic altruism might be a better approximation.<sup>15</sup>

# E. Investment in Future Generations Without BIGs

This section shows that BIGs are not the only type of exchange that can be used to sustain FIGs. To see this, consider the following generalization of the model. Every period t, generation t - 1 makes two choices: (1) how much to invest in FIGs, just like before, and (2) how many resources to spend in another good. Denote the second decision by  $e_t$ . The preferences of generation t - 1 are now given by

(21) 
$$U(c^{y}, c^{m}, c^{o}) + F(f_{t-(k+1)}) + B((e_{k})_{k=t-1,t,t+1, \dots}).$$

Note that a generation could be affected by all the choices made in the non-FIG dimension after it is born. This includes, as special cases, the previous model of BIGs, and the case in which the non-FIG dimension only affects those alive at the time.

Let  $e^{au}$  denote the action taken in generational autarky, and  $e^*$  the optimal stationary action. Suppose that  $e^{au} \neq e^*$  so that cooperation is required to sustain  $e^*$ . For any path  $\varepsilon =$  $\{\hat{e}_t\}_{t=1}^{\infty}$  define the continuation surplus function

(22) 
$$S_t^E(\varepsilon) \equiv [V(\bar{w} - \hat{e}_t) + B((\hat{e}_k)_{k=t-1,t,t+1,...})] - [V(\bar{w}) + B(e^{au}, e^{au}, ...)],$$

that measures the value of the allocation  $\varepsilon$  over generational autarky. It is straightforward to extend the previous arguments to show that a positive amount of investment in FIGs can be sustained if and only if there exists a path  $\varepsilon$  such that  $S_t^E(\varepsilon) > 0$  for all t.

The intuition is simple. The central feature of FIGs is that present generations do not care about how many FIGs are produced by future decision makers. This is what makes FIGs difficult to sustain, and the reason a link is needed. If there exists another dimension of exchange in the organization that requires cooperation, and if the cooperative allocation generates a positive continuation surplus, then that surplus can be used to sustain investment in FIGs using strat-

<sup>&</sup>lt;sup>15</sup> See Joseph Altonji et al. (1992, 1997).

egies that link FIGs and non-FIGs. It does not matter what the other dimension of exchange is as long as: (1) it requires IG cooperation, and (2) it generates a positive continuation surplus. For this to be the case, present decision makers must care about future decisions.

This argument can be pushed even further. In organizations in which more than two decisions are made, say if there are several BIGs, it is possible to link several of these decisions to FIGs. If each individual BIG generates a surplus, each additional BIG provides additional incentives to provide FIGs. In the context of the political economy of IG public goods discussed in Section V, pay-as-you-go social insurance, the choice of capital tax rates, and the decision to honor the national debt can be used simultaneously to sustain investment in IG public goods.

#### **IV. The Effect of Mandatory Provision**

The previous analysis has shown that there are always equilibria in which FIGs and/or BIGs are not provided. A natural question to ask is whether the introduction of minimal provision constraints can improve the outcomes generated by these organizations. These types of constraints are common. For example, taxfinanced public education places a constraint on the minimum amount of educational expenditures that a parent can give to his child, and mandatory old age social insurance imposes a similar constraint for the case of BIGs.<sup>16</sup>

In this section we study the effect that minimal provision constraints have on voluntary provision. Denote the minimum constraint by  $(\underline{b}, f)$ . These constraints have no effect on the payoffs, but shrink the action sets to  $\{(b, f)|b + f \leq \overline{w}, f \geq f, \text{ and } b \geq \underline{b}\}$ .

To characterize the equilibrium set for this case we need to define a new continuation surplus function. Let  $\gamma$  be any path of BIGs and FIGs satisfying the constraints and define

(23) 
$$S_t^{(\underline{b},\underline{f})}(\gamma) \equiv \left[ V(\overline{w} - \hat{b}_t - \hat{f}_t) + B(\hat{b}_{t+1}) \right] - \left[ V(\overline{w} - \underline{f} - \underline{b}) + B(\underline{b}) \right].$$

It is straightforward to extend the arguments in Propositions 1 and 2 to show that a path  $\gamma$  can be sustained as an equilibrium if and only if  $S_t^{(\underline{b},\underline{f})}(\gamma) \ge 0$  for all *t*. This provides a full characterization of the equilibrium set. The only difference is that now the payoff of generational autarky has changed to  $V(\overline{w} - \underline{f} - \underline{b}) + B(\underline{b})$ , which has implications for the levels of BIGs and FIGs that can be sustained.

Let  $f^{\max}(\underline{b}, \underline{f}), f^{\min}(\underline{b}, \underline{f}), b^{\max}(\underline{b}, \underline{f})$ , and  $b^{\min}(\underline{b}, \underline{f})$  denote, respectively, the maximum and minimum level of FIGs and BIGs that can be sustained as a stationary equilibrium in an institution with constraints  $(\underline{b}, \underline{f})$ . The following proposition describes the effect of (1) introducing a minimum constraint only on BIGs ( $\underline{f} = 0$ ), and (2) introducing a minimum constraint only on FIGs ( $\underline{b} = 0$ ). The effect of introducing a constraint in both goods is discussed below.

PROPOSITION 6: Suppose that B'(0) > V'(0), then:<sup>17</sup>

- (1) The introduction of a minimum provision constraint  $\underline{b} \in [0, \overline{w})$  on BIGs has the following effects:  $f^{\max}(\underline{b})$  decreases as  $\underline{b}$ increases between 0 and  $b_B^*$  and equals 0 afterwards,  $f^{\min}(\underline{b}) = 0$  for all  $\underline{b}$ ,  $b^{\max}(\underline{b})$ decreases between 0 and  $b_B^*$  and equals  $\underline{b}$ afterwards, and  $b^{\min}(\underline{b}) = \underline{b}$ .
- (2) The introduction of a minimum provision constraint  $f \in [0, \bar{w})$  on FIGs has the following effects:  $f^{\max}(f)$  is increasing,  $f^{\min}(f) = f$ ,  $b^{\max}(f)$  is decreasing, and  $b^{\min}(f) = 0$ .

Figure 3 summarizes the effects described in this result. Figures 4 (left-hand side) depicts the effect of  $\underline{b}$  on the equilibrium set. For  $\underline{b} \in [0, b_B^*]$  the set shrinks towards  $(b_B^*, 0)$  as  $\underline{b}$  increases,  $b^{\min}(\cdot)$  increases, and  $b^{\max}(\cdot)$  decreases. The intuition for the latter effect follows from

<sup>17</sup> This condition can be relaxed to  $B'(0) > V'(\bar{w})$ . But in that case there may exist  $\hat{f}$  such that, for all  $f > \hat{f}$ ,  $f^{\max}(\underline{f}) = f^{\min}(\underline{f})$  and  $b^{\max}(\underline{f}) = b^{\min}(\underline{f}) = 0$ .

<sup>&</sup>lt;sup>16</sup> Another natural institution is a constitutional constraint requiring the provision of exactly the optimal level of BIGs and FIGs in every period. This institution, however, does not work well when there is considerable uncertainty about future parameters. For example, what will be the optimal amount of R&D in nanotechnology in 2050?

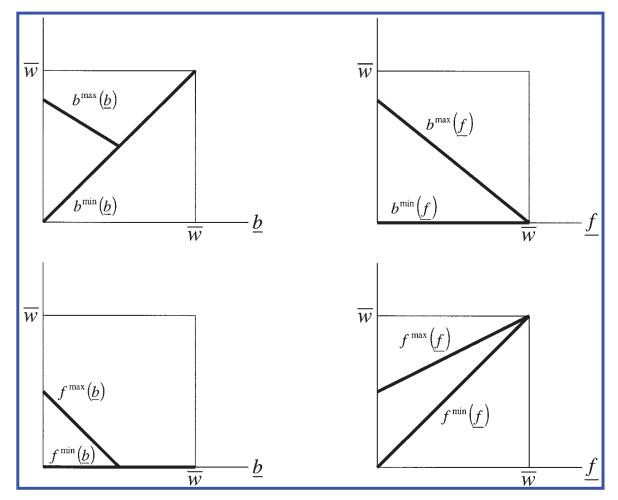


FIGURE 3. EFFECT OF MINIMAL PROVISION CONSTRAINTS ON MAXIMAL AND MINIMAL PROVISION

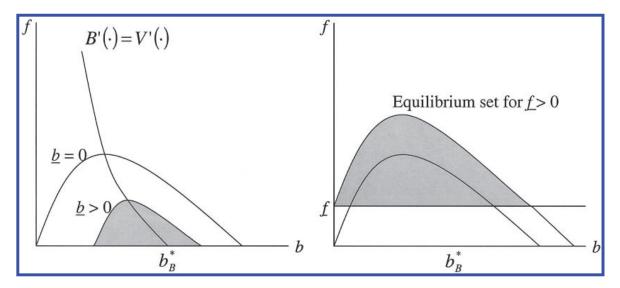


FIGURE 4. EFFECT OF MINIMAL PROVISION CONSTRAINTS ON THE SET OF STATIONARY EQUILIBRIUM OUTCOMES

the fact that the payoff at generational autarky, which is the worse feasible punishment for a generation that fails to cooperate, increases in this range. This reduces the incentives to cooperate. At  $\underline{b} = b_B^*$ , the equilibrium set shrinks to a single point:  $(b_B^*, 0)$ . Further increases in  $\underline{b}$ generate the equilibrium set  $\{(\underline{b}, 0)\}$ . Since the BIG surplus is needed to sustain FIGs, the maximum level of FIGs that can be sustained decreases with  $\underline{b}$ . In fact, by the time  $\underline{b}$  reaches  $b_B^*$ , the BIGs generate no surplus and thus no FIGs can be sustained.

This result shows that there is a perverse institutional trade-off in this class of organizations. The imposition of a minimal provision constraint in BIGs eliminates the possibility of the bad equilibrium in which no BIGs are produced, but it also reduces the maximum level of investment in future generations that can be sustained.<sup>18</sup>

The constraint for FIGs f has a different effect. Increases in f have a positive effect on the maximum and minimum level of FIGs that can be sustained, and a negative effect on the maximum amount of BIGs. The result is also driven by the effect of f on generational autarky payoff, given by  $V(\bar{w} - f) + B(0)$ , which is decreasing in f. As a result, as shown in Figure 4 (right-hand side), bundles (b, f) that were not sustainable before, now become sustainable.

Now consider the effect of introducing a minimum provision constraint in both FIGs and BIGs. Since the analysis is very similar to the previous two cases, the details are omitted. Once more, the impact on FIG provision depends on the payoff at generational autarky. If  $V(\bar{w} - \underline{b} - f) + B(\underline{b}) > V(\bar{w}) + B(0)$  the equilibrium set with constraints is a subset of the equilibrium set for the case of no constraints, and is similar to the one depicted in Figure 4 (left-hand side) except for one minor change: the points (b, f) for which f < f have to be removed. As a result, the constraint decreases the maximum level of FIGs that can be sustained. By contrast, if  $V(\bar{w} - \underline{b} - \underline{f}) + B(\underline{b}) < V(\bar{w}) + B(0)$ , the equilibrium set resembles the one depicted in Figure 4 (right-hand side) with one minor change: the points (b, f) for which  $b < \underline{b}$  have to be removed from the set. In this case, the maximum level of FIGs that can be sustained increases.

Minimal provision constraints for BIGs and FIGs have very different effects. Whereas increases in the constraint for BIGs crowd out voluntary cooperation in BIGs and FIGs, increases in the constraint for FIGs increase the total amount of FIGs that can be sustained. This asymmetry is interesting because, to the extent that the minimal provision constraints are determined endogenously, present generations have an incentive to introduce minimal provision constraints in BIGs but not on FIGs.

#### **V.** Applications

#### A. Investment in Children Within the Family

A natural interpretation of the model is as a theory of IG exchange within a family that is either selfish or exhibits paternalistic altruism. Families exchange two types of IG goods: (1) FIGs, that are provided by parents to their young children in the form of education and parental care, and (2) BIGs, that adult children provide to their aging parents in the form of care, insurance, and status. In this interpretation k = 0.

As shown in Section III, subsections B and D, in a selfish family investment in children can be positive only if there is a link between BIGs and FIGs, and in families with paternalistic altruism the link is required to generate investments in excess of what parents are willing to invest on their own. For example, without BIGs, a parent might be willing to finance a high school education, but not college. In both types of families, at least part of the investments are driven by strategic considerations: parents believe that (excess) investments in FIGs are the price that they pay for getting the BIGs that they desire in old age.

The model predicts the existence of three types of families. First, families that link BIGs and FIGs sustain a high level of investment in

<sup>&</sup>lt;sup>18</sup> Bernheim and Whinston (1998) obtain results with a similar flavor for nonintergenerational contracting problems. In many economic relationships complete contracts are impossible. In this case, voluntary cooperation is needed in the dimensions where the contract is incomplete. In this context, they show that it might be advantageous to purposefully leave some dimensions out of the contract to increase the incentive to cooperate in the dimensions that cannot be included.

both. Second, gerontocratic families with high levels of provision for the elderly, but low investment in children. Third, "dysfunctional" families that underprovide both. Casual observation suggests that there is significant variation within and across cultures. Endogenous preferences are likely to be an important part of the explanation, specially if culture and institutions influence the amount of altruism within families. But family norms might also play an important role: some societies and families converge to cooperative codes of behavior, others do not.

As shown in Section III, in order for FIGs to be provided, there must be a BIG that generates a positive surplus. This is guaranteed as long as the elderly place a marginal value on the first unit of the BIG that exceeds the marginal cost for the middle-aged. Although in our stylized model this is imposed as an assumption [condition  $B'(0) > V'(\bar{w})$  in Proposition 3], several of the BIGs exchanged within families satisfy these characteristics. Consider, for example, the case of insurance. Retirees face risks that are not insurable through financial markets such as a collapse of the stock market, or a crime that significantly reduces their wealth. As long as the serial correlation of the adverse shocks is low, there are gains from exchange between the middle-age and the elderly. Kotlikoff and Avia Spivak (1981) show that a similar argument holds for the provision of "annuity insurance" within the family when annuity markets are imperfect.

The results on mandatory provision also have interesting implications for the family. Suppose that the government introduces a law that forces the middle-aged to provide a BIG that they used to provide voluntarily. This would be the case, for example, if the good is financed with taxes on the middle-aged. This has three types of effects (see Figure 3). First, it increases the minimal amount of BIGs that any elderly person receives. If a fraction of the families in the economy are in a bad equilibrium in which BIGs are not provided, the policy improves the welfare of these elderly. Second, it decreases the maximum amount of voluntary provision of BIGs that can be sustained. Thus, if the public program is not large enough to fully crowd out family care, the welfare of the elderly who belong to families that are in a good equilibrium

can go down.<sup>19</sup> Finally, it reduces the maximum amount of FIGs that can be sustained, and thus can crowd out investment in children within the family.<sup>20</sup> These mechanisms could contribute to our understanding of some trends that have taken place in the last few decades of the twentieth century: (1) an increase in the generosity of government transfers to the elderly, (2) an increase in measures of family disintegration, (3) a decrease in the amount of time that parents spend with their children, (4) a decrease in educational performance, and (5) a decrease in the birth rate.

# B. Why Is Social Security Good for the Environment?

Now consider the political economy of IG exchange. In this case the FIG is an IG public good, such as the environment or R&D, and the BIG is a pay-as-you-go social insurance program such as social security or Medicare.

This application requires a slight specialization of the model. The key difference is that now decisions are made by majority rule, and thus many agents participate in the decisionmaking process. Also, to further explore the nature of BIGs, we model the social security program explicitly, and consider a more realistic demographic structure.<sup>21</sup> As we will see, the basic insights remain unchanged.

<sup>19</sup> Whether or not there is a reduction on the total level of BIGs consumed by this type of families depends on the equilibrium that was played originally. As can be seen from Figure 4, a minimum provision constraints eliminates some but not all equilibria. A similar comment applies to the next statement.

<sup>21</sup> Another reason for complicating the demographic structure is that with voting the three-period model is a knife-edge case. It generates results that do not hold as long as agents can live for five or more periods.

<sup>&</sup>lt;sup>20</sup> Of course, this assumes that the government intervenes only in BIGs. One could argue that the problem disappears if the government intervenes in both BIG and FIGs. But, at best, this can only be a partial solution. First of all, government programs require revenue that must be raised with costly distortionary taxes. Second, these programs are not likely to be tailored optimally to the specific needs of each family. Finally, and perhaps most importantly, the nature of some BIGs and FIGs is such that they can only be provided in the context of the family. For example, there does not seem to be a substitute for the impact that parents' care and love have on the emotional and character development of children.

Agents live for nine periods, each period representing a decade of life. They are dependent children in the first two periods, workers in the next five, and retirees in the last two. There is no population growth. Workers receive a wage w; everyone else has no income. Agents can borrow and save at the constant interest rate r > 0, which implies that the economy is dynamically efficient.

Every period society needs to choose the size of a balanced pay-as-you-go social security system. Let  $T_t^s$  denote the lump-sum payroll tax paid by workers in period t, and  $B_t^s = \frac{5}{2}T_t^s$ denote the benefits for retirees.<sup>22</sup> Society also chooses the level  $E_t$  of expenditures in a FIG that benefits children and future generations. The revenue for the FIG is raised using equal lump-sum taxes on the workers and elderly. Preferences are now given by  $U(c^1, \dots, c^9) + F(\{E_{t-k}\}_{k\leq -1})$ . Note that the social security BIG is a transfer, instead of a commodity that enters directly in the utility function.

Consider a social security system  $T^s = \{\hat{T}_t^s\}_{t=1}^{\infty}$ . The continuation value of the system for generation *t*, at age *a*, is given by

(24) 
$$CV_{t}^{a}(T^{s}) = \sum_{k=8}^{9} \frac{\hat{B}_{t+k-1}^{s}}{(1+r)^{k-a}} - \sum_{k=a}^{7} \frac{\hat{T}_{t+k-1}^{s}}{(1+r)^{k-a}},$$

for a worker of age *a*, and

(25) 
$$CV_t^a(T^s) = \sum_{k=a}^9 \frac{\hat{B}_{t+k-1}^s}{(1+r)^{k-a}}$$

for a retiree of age a = 8, 9. At its name indicates,  $CV_t^a(\cdot)$  measures the value for an agent of age *a* of keeping the social security system going. The continuation value will play a role analogous to the surplus generated by BIGs in the standard model: investment in FIGs can be sustained only as long as the continuation value is positive for a *majority* of voters in every period.

To simplify we discretize the policy space. Let  $\gamma = \{(\hat{T}_t^s, \hat{E}_t)\}_{t=1}^{\infty}$  denote a path for fiscal policy. We assume that every period society can only choose between two levels of the BIG,  $T_t^s \in \{0, \hat{T}_t^s\}$ , and two levels of the FIG,  $E_t \in \{0, \hat{E}_t\}$ .

Every adult agent  $(a \ge 3)$  casts a vote  $\sigma_a^s(h_t) \in \{0, \hat{T}_t^s\}$  for social security, and a vote,  $\sigma^f(h_t) \in \{0, \hat{E}_t\}$  for the FIG.  $h_t$  denotes the history of policy outcomes. Individual votes are not observable. Decisions are made by issueby-issue direct democracy: in each dimension, the level chosen by the majority is implemented.<sup>23</sup> However, to deal with the well-known problem of indifference among nonpivotal voters, we assume as-if-pivotal-voting: agents always cast their vote for the option that yields their preferred continuation outcome. This refinement rules out unreasonable equilibria in which agents cast votes against their preferences.

**PROPOSITION 7:** 

- (1) A path  $\gamma$  with a positive level of social security benefits can be sustained as an equilibrium as long as  $CV_t^a(\hat{T}^s) \ge 0$  for all a = 6, ..., 9 and t.
- (2) A path γ with a positive level of investment in FIGs can be sustained as long as the following conditions are true:
- (i)  $CV_t^a(\hat{T}^s) \ge (\hat{E}_t/7) \ge 0$  for all a = 5, ..., 8 and t.
- *(ii) agents play voting strategies that link social security and FIGs.*

The first part of the result provides necessary and sufficient conditions for social security to be sustainable by majority rule. As before, it is useful to focus first in the case in which there are only BIGs. For the purpose of building intuition consider stationary policies in which  $\hat{T}_t^s = \hat{T}^s$  for all t. Since the economy is dynamically efficient, the continuation value of social security for sufficiently young voters is

<sup>&</sup>lt;sup>22</sup> The benefit formula follows from the fact that there are five workers for every two retirees.

<sup>&</sup>lt;sup>23</sup> The results below hold for any political institution in which Condorcet winners are selected whenever they exist.

negative. Let  $\bar{a}(T^s)$  denote the smallest age at which the continuation value becomes positive. Workers with  $a < \bar{a}(T^s)$  always vote against social security. Similarly, retirees always vote for the system since they do not have to pay more payroll taxes. This implies that social security's fate depends on the vote of the middleaged group, with ages between  $\bar{a}(T^s)$  and 7. Since seven generations (a = 3, ..., 9) cast a vote every period, the system passes with at least four votes as long as the middle-aged vote positively and  $\bar{a}(T^s) \leq 6$ . Note that the middleaged vote for social security not because they care about current retirees, but because they correctly believe that otherwise they will not be able to receive benefits.

The second part of the result provides necessary and sufficient conditions for a positive level of investment in FIGs to be sustained by majority rule. The forces at work are also similar to the ones for the basic model. First, present voters care about social security but not about FIGs such as the environment. Thus, they vote against the environment unless future voters play a voting strategy that links BIGs and FIGs: a generation is punished in retirement if, during its voting years, society failed to provide social security for its parents or to invest sufficiently in FIGs. Note, however, that the punishment is conditioned on the outcome of the election, and not on the voting behavior of particular generations, since individual votes are not observable. Second, there is a limit to how much investment in BIGs can be sustained. A generation is willing to vote for FIGs only if the taxes that it has to pay,  $(\hat{E}_t/7)$ , are less than the continuation value of the system,  $CV_t^a(\hat{T}^s)$ .

As before, the benefits that the FIGs generate on future generations play no role on their sustainability, only the direct benefits for the generations making the investment do. This implies that programs such as the Clean Air Act, which generate benefits in the short term, are more likely to be financed than programs like global warming prevention, where most of the benefits appear only in the very long run. In particular, there could be programs that generate much larger benefits in the long run, and have a better social rate of return, but that are not produced because those benefits only accrue to unborn generations.

Bohn (1998) has studied the political econ-

omy of pay-as-you-go social insurance in the United States. He calculates the continuation value of social security for voters of different ages and shows that it is negative for young voters, but strictly positive for voters at or above the median age. As a result, social security is sustainable and it generates surplus that can be used to sustain investments in the environment. This is the reason why social security can be good for the environment.

Other public BIGs that could be used to sustain FIGs include the choice of capital tax rates and the decision to honor the national debt. Consider the first example. Every generation needs to save for retirement and can do so only if future generations refrain from expropriating its savings. However, every generation would like to expropriate the current elderly through a 100-percent capital tax, but not to be expropriated in old age. In this case producing the BIG takes the form of selecting a low capital tax for the current period.

Propositions 3 and 7 provide a different perspective on the political economy of logrolling. The prevailing view in the literature is that logrolling is often a cause of inefficiencies because it allows inefficient "pork barrel" projects to be enacted.<sup>24</sup> One can think of the link between BIGs and FIGs as an IG and dynamic form of logrolling in which agents who favor the environment are willing to vote for social security, but only if current retirees invested in future generations when they were young. This form of logrolling is beneficial since it is essential to sustain investment in future generations.

The results on mandatory provision have interesting political economy implications. Consider a constitutional reform requiring that a sufficiently large minimum level of social security benefits be paid every period unless a supermajority votes against it. If the supermajority requirement is strong enough, the reform gives veto power to the elderly, who always vote for social security. In this case, middle-aged workers know that social security cannot be voted down and thus have no incentive to invest in FIGs. Similarly, some analysts have proposed eliminating the current system and moving to a system of personal savings accounts. If expropriation of the balances in these accounts

<sup>&</sup>lt;sup>24</sup> For example, see Gordon Tullock (1998).

through taxation is very unlikely (perhaps because of constitutional or other legal restrictions), the retirement benefits of current workers do not depend on the actions of future generations. This eliminates the need for IG cooperation in social security, and thus a source of surplus that could be used to sustain investment in future generations.

A surprising implication of the model is that the aging of the electorate can be beneficial for future generations. To see this, consider a small complication of the political economy model in which, for exogenous reasons, only a fraction of the population in each age group votes.<sup>25</sup> Proposition 7 then changes as follows. First, a pay-asyou go social security system is sustainable as long as it generates a positive continuation value for a majority of the population that *actually* votes. Second, a positive level of FIGs can be sustained as long as if  $CV_t^a(\hat{T}_t^s) \ge (\hat{E}_t/7)$  for a majority of the population that *actually* votes.

Suppose, for the purposes of this example, that the continuation value of social security becomes positive at age 6. If the share of agents that vote is constant across age groups, social security can be sustained but FIGs cannot (they only get three votes, those of ages 6 to 8, since the eldest always vote against FIGs). Compare this with a situation in which voters in ages 6 to 9 become twice more likely to vote than younger voters in ages 3 to 5. In this case, the voters in ages 6 to 8 constitute a majority and investment in FIGs can be sustained. Intuitively, any demographic change that increases the continuation surplus of the "median voter" increases the amount of FIGs that can be sustained.

# **VI. Concluding Remarks**

This paper has studied the ability of nonmarket institutions, such as the government and the family, to invest optimally in future generations. We have shown that BIGs, such as social security, play a crucial role in sustaining investment in FIGs: without them, investment in FIGs is inefficiently low; with them, even optimal investment by selfish generations is possible.

We have shown that IG organizations can con-

verge to three types of equilibria: underprovision of BIGs and FIGs, provision of BIGs but not FIGs, and provision of both. This multiplicity of equilibria has normative and positive implications.

From a normative point of view, the multiplicity represents an opportunity. The link between BIGs and FIGs is a mechanism that could be harnessed by institutional designers to sustain investment in future generations. Consider, for example, the introduction of a constitutional constraint that requires a minimal amount of expenditure in FIGs for every dollar spent on the elderly. This constraint forces a link between BIGs and FIGs analogous to the one that arises in the equilibrium with positive investment in future generations. As long as the required amount of FIG expenditures do not exceed the surplus that the "median voter" gets from pay-as-you-go social insurance, the reform kills the bad equilibrium in which BIGs are provided but FIGs are not.<sup>26</sup> Another potentially useful reform would make the link between BIGs and FIGs more transparent by requiring legislation in BIGs and FIGs to be debated and voted on together, as a package.

From a positive point of view, the multiplicity of equilibria calls for empirical study. Casual observation suggests that the political process has not coordinated to the "good equilibrium," since social security and the environment do not seem to be linked in the public debate. However, the link might play an important role in other organizations such as the family. After all, the quid pro quo nature of intergenerational exchange is more transparent within a family than at the social level. Parents seem to understand that their behavior towards their children influences their emotional development and how they are treated in old age. By contrast, it might be more difficult for a voter to understand that present policies could affect the voting attitudes of future generations. Nevertheless, given the lack of a solid theoretical foundation for choosing one equilibrium over another, the best the theory can do for now is characterize the entire set of possible organizational outcomes, and provide guidelines for how to bring the theory to the data.

<sup>&</sup>lt;sup>25</sup> Say, if agents have heterogeneous preferences for political participation and/or heterogeneous costs of voting.

<sup>&</sup>lt;sup>26</sup> See Rangel (2002) for an institution that generates the link using market forces.

#### APPENDIX

PROOF OF PROPOSITION 1:

Sufficiency is obvious, now look at necessity. Consider any path  $\gamma = \{(\hat{b}_t, \hat{f}_t)\}_{t=1}^{\infty}$  that can be sustained as a subgame-perfect equilibrium. It must be the case that, for all t,

(A1) 
$$V(\bar{w} - \hat{b}_t - \hat{f}_t) + F(\hat{f}_{t-(k+1)}) + B(\hat{b}_{t+1}) \ge V(\bar{w}) + F(\hat{f}_{t-(k+1)}) + B(0).$$

This follows because  $(\hat{b}_t, \hat{f}_t)$  must be a best response along the equilibrium path. If this inequality is violated, generation t - 1 is better off choosing (0, 0) in history  $h_t = ((\hat{b}_1, \hat{f}_1), \dots, (\hat{b}_{t-1}, \hat{f}_{t-1}))$ —a contradiction.

Let  $s_t(h_t) = (s_t^B(h_t), s_t^F(h_t))$  be the STSs associated with  $\gamma$ . Since the STSs generate  $\gamma$  as the outcome path, it suffices to show that they are a subgame-perfect equilibrium. We need to consider two types of histories.

(i)  $\mu(h_t) = C$ . In this case, generation t - 1 plays  $s_t(h_t)$  if and only if

$$V(\bar{w} - \hat{b}_t - \hat{f}_t) + F(f_{t-(k+1)}) + B(\hat{b}_{t+1})$$
  

$$\geq \underset{(b,f) \neq (\hat{b}_t, \hat{f}_t)}{\arg \max} V(\bar{w} - b - f) + F(f_{t-(k+1)}) + B(s_{t+1}^B(h_t, (b, f)))$$
  

$$= V(\bar{w}) + F(f_{t-(k+1)}) + B(0).$$

The equality follows because (0, 0) is the best possible deviation. The claim then follows from (A1).

(ii)  $\mu(h_t) = P$ . Here, generation t - 1 plays  $s_t(h_t)$  if and only if

$$V(\bar{w} - \hat{f}_t) + F(f_{t-(k+1)}) + B(\hat{b}_{t+1})$$
  

$$\geq \underset{(b,f) \neq (\hat{b}_t, \hat{f}_t)}{\arg \max} V(\bar{w} - b - f) + F(f_{t-(k+1)}) + B(b_{t+1}(s_t^B, (b, f)))$$
  

$$= V(\bar{w}) + F(f_{t-(k+1)}) + B(0).$$

Once more, (0, 0) is the best possible deviation. Equation (A1) implies that the inequality is satisfied.

# **PROOF OF PROPOSITION 2:**

By Proposition 1, it suffices to show that the STSs associated with  $\gamma$  are satisfied if and only if (7) is satisfied. Let  $s_t(h_t) = (s_t^B(h_t), s_t^F(h_t))$  be the STS associated with  $\gamma$ . For this strategy profile to be a subgame-perfect equilibrium it must be the case that, for all histories  $h_t$  with  $\mu(h_t) = C$ ,

$$V(\bar{w} - \hat{b}_t - \hat{f}_t) + F(f_{t-(k+1)}) + B(\hat{b}_{t+1})$$
  

$$\geq \underset{(b,f) \neq (\hat{b}_t, \hat{f}_t)}{\arg \max} V(\bar{w} - b - f) + F(f_{t-(k+1)}) + B(s_{t+1}^B(h_t, (b, f)))$$
  

$$= V(\bar{w}) + F(f_{t-(k+1)}) + B(0).$$

Similarly, for the STS to be an equilibrium, it must be the case that for all histories  $h_t$  with  $\mu(h_t) = P$ ,

$$V(\bar{w} - \hat{f}_t) + F(f_{t-(k+1)}) + B(\hat{b}_{t+1})$$
  

$$\geq \underset{(b,j) \neq (\hat{b}_t, \hat{f}_t)}{\arg \max} V(\bar{w} - b - f) + F(f_{t-(k+1)}) + B(s_{t+1}^B(h_t, (b, f)))$$
  

$$= V(\bar{w}) + F(f_{t-(k+1)}) + B(0).$$

If (7) is satisfied, then these two conditions hold and the STS is a subgame-perfect equilibrium. If (7) is violated, the first condition cannot hold and the STS is not an equilibrium.

# **PROOF OF PROPOSITION 3:**

- (1) Follows directly from Proposition 2.
- (2) Follows from part 1 plus the fact that  $S_t(\gamma) < S_t^B(\gamma)$  whenever  $\hat{f}_t > 0$ . (3)  $B'(0) > V'(\bar{w})$  implies that  $S_{st}^B(b) > 0$  for some b > 0. The result then follows from (2).

### **PROOF OF PROPOSITION 5:**

- (1) This part of the proof is very similar to the proof of Propositions 1 and 2 and thus is omitted.
- (2) Consider any stationary equilibrium in which BIGs and FIGs are not linked. That means that, for all t, generation t 1 plays a strategy of the form  $s_t(h_t) = (s_t^B(b_1, \dots, b_{t-1}), s_t^F(f_1, \dots, f_{t-1}))$ . Let  $\tilde{b}$  denote the level of BIGs generated by such a strategy. Clearly,  $\tilde{b} \leq b_B^{\max}$ , the maximum level of BIGs that can be sustained when there are only BIGs. Since generation t 1 is not affected by decisions about FIGs taken after period t, it does not care about how future agents respond to its choice of  $f_t$ . Thus, along the equilibrium path, every generation solves

$$\max_{f} V(\bar{w} - \tilde{b} - f) + B(\tilde{b}) + G(f).$$

Then,  $G'(0) > V'(\bar{w} - b_B^{\max})$  implies that the equilibrium level of FIGs must be  $\tilde{f}$  satisfying  $V'(\bar{w} - \tilde{b} - \tilde{f}) = G'(\tilde{f})$ .

To conclude the proof, consider a marginal increase in the production of FIGs in every period. The impact on the utility of generations born after period k + 1 is given by

$$-V'(\bar{w}-\tilde{b}-\tilde{f})+G'(\tilde{f})+F'(\tilde{f})>0.$$

The impact on generations 0 to k - 1 is given by

$$-V'(\bar{w}-\tilde{b}-\tilde{f})+G'(\tilde{f})=0.$$

**PROOF OF PROPOSITION 6:** 

(1) Consider first the introduction of a minimum provision constraint  $\underline{b} \in [0, \overline{w})$  on BIGs. The set of bundles (b, f) that can be sustained as a stationary equilibrium is given by:

$$\{(b, f) | V(\bar{w} - b - f) + B(b) \ge V(\bar{w} - b) + B(b), f \ge 0, \text{ and } b \ge b\}.$$

(The proof of this step is almost identical to the proofs of Propositions 1 and 2 and is therefore omitted.)

This set satisfies the following properties: (1) for  $\underline{b} \in [0, b_B^*]$ , the set shrinks with  $\underline{b}$  as depicted in Figure 4 (left-hand side); and (2) for  $\underline{b} \in [b_B^*, \overline{w})$ , the set equals  $\{(\underline{b}, 0)\}$ . Let  $\phi_{\underline{b}}(b)$  denote the level of FIGs that defines the upper boundary of the equilibrium set. This boundary is implicitly defined by  $V(\overline{w} - b - \phi) + B(b) = V(\overline{w}) + B(\underline{b})$ . By the IFT,  $\phi_{\underline{b}}(b)$  is a

continuously differentiable function, and  $\phi'_b(b) = \frac{B' - V'}{V'}$ . Thus, the boundary is increasing to the left of the locus  $\{(b, f) | B'(b, f) = V'(b, f)\}$ , and decreasing to the right. Given that  $B'(0) > V'(\bar{w})$ , this locus intersects the horizontal axis at  $b^*_B > 0$  and has the shape depicted in Figure 4. The properties of  $f^{\max}$ ,  $f^{\min}$ ,  $b^{\max}$ , and  $b^{\min}$  then follow directly.

(2) Now consider the introduction of a minimum provision constraint  $f \in [0, \bar{w})$  on FIGs. The set of bundles (b, f) that can be sustained as a stationary equilibrium is given by:

$$\{(b,f) | V(\bar{w} - b - f) + B(b) \ge V(\bar{w} - f) + B(0), b \ge 0, \text{ and } f \ge f\}.$$

(The proof of this step is also almost identical to the proofs of Propositions 1 and 2 and is omitted.)

Consider the equilibrium set defined by f which is depicted in Figure 4 (right-hand side). It is easy to see that  $f^{\min}(f) = f$ , and  $b^{\min}(f) = 0$ . Let  $\phi_f(b)$  denote the level of FIGs that define the upper boundary of the set, which is implicitly defined by  $V(\bar{w} - b - f) + B(b) =$  $V(\bar{w} - f) + B(0)$ . By the IFT,  $\phi_f(b)$  is a continuously differentiable function and  $\phi'_f(b) = \frac{B' - V'}{V'}$ . B'(0) > V'(0) implies that  $\phi'_f(0) > 0$  for all  $f \in [0, \bar{w})$ . Also, since  $\phi_f(0) = f$ , we get that,  $f^{\max}(f) > f^{\min}(f)$ . Finally, as shown in the figure, since the autarky payoff  $V(\bar{w} - f) + B(0)$  is strictly decreasing in f,  $f^{\max}(f)$  must be increasing in f.

Finally look at  $b^{\max}(f)$ , which is defined implicitly by the equation

$$V(\bar{w} - b - f) + B(b) = V(\bar{w} - f) + B(0).$$

By the IFT,  $b^{\max}(f)$  is continuously differentiable with

$$\frac{\partial b^{\max}(\underline{f})}{\partial \underline{f}} = \frac{V'(\bar{w} - b^{\max}(\underline{f}) - \underline{f}) - V'(\bar{w} - \underline{f})}{-V'(\bar{w} - b^{\max}(\underline{f}) - \underline{f}) + B'(b^{\max}(\underline{f}))},$$

The sign of this derivative is equal to the sign of the denominator. But since  $b^{\max}(\underline{f})$  occurs to the right of the locus  $\{(b, f)|B'(b, f) = V'(b, f)\}$ , the sign is negative.

# PROOF OF PROPOSITION 7:

(1) Let  $\gamma = \{(\hat{T}_t^s, \hat{E}_t)\}_{t=1}^{\infty}$  be a policy path in which there are no FIGs  $(\hat{E}_t = 0 \text{ for all } t)$  and in which payroll taxes are positive in every period. For every period t, let  $\bar{a}_t(T^s)$  denote the smallest age at which the continuation value of social security becomes positive. Consider the following voting strategies for any period t:

(A2) 
$$\sigma_a^s(h_t) = \hat{T}_t^s \text{ for } a = 8, 9;$$

(A3) 
$$\sigma_a^s(h_t) = 0 \text{ for } a = 3, ..., \bar{a}_t(T^s) - 1; \text{ and}$$

(A4) 
$$\sigma_a^s(h_t) = \begin{cases} \hat{T}_t^s & \text{if } T_k^s = \hat{T}_k^s \text{ for all } k < t \text{ or } t = 1\\ 0 & \text{otherwise} \end{cases} \text{ for } a = \bar{a}_t(T^s), \dots, 7.$$

We claim that if  $\bar{a}_t(T^s) \le 6$  for all t, these strategies are an equilibrium in which social security is implemented every period. We need to consider two types of histories: those in which social security has always won  $(T_k = \hat{T}_k^s \text{ for all } k < t)$ , and those in which it has been defeated at least once in the past.

Consider the second type of histories first. Given (A2) to (A4), everyone knows that social

security will not exist in the future. As a result, the best response of the workers is to vote for 0 payroll taxes, and the best response of the retirees is to vote for  $\hat{T}_t^s$ .

Now consider the first type of histories. Clearly, retirees must always vote as in (A2) since they benefit from the positive payroll taxes. Voters in ages  $a = \bar{a}_t(T^s), ..., 7$  know that if social security wins, it will be there for the rest of their lives, and that if it is defeated the system collapses forever. Since they have a positive continuation value, they vote for  $\hat{T}_t^s$ . For an analogous reason, voters younger than  $\bar{a}_t(T^s)$  always vote for 0. As long as  $\bar{a}_t(T^s) \leq 6$ ,  $\hat{T}_t^s$  wins with four or more votes.

(2) Let  $\gamma = \{(\hat{T}_t^s, \hat{E}_t)\}_{t=1}^{\infty}$  be a policy path in which  $\hat{E}_t > 0$  for all t. Let  $h_t = \{(T_k^s, E_k)\}_{k=1}^{t-1}$  denote the history of public policy. Suppose that agents play voting strategies of the form  $\sigma_a(h_t) = (\sigma_a^s(T_1^s, \dots, T_{t-1}^s), \sigma_a^f(h_t))$ . In this case agents know that their vote on FIGs have no effect on how future generations care about BIGs. Since they do not benefit from future FIGs, their optimal response must be to vote for zero FIG provision. Thus, no FIGs are provided.

To show that  $\gamma$  can be sustained as long as  $CV_t^a(\hat{T}_t^s) \ge (\hat{E}_t/7) \ge 0$  for all t and a = 5, ..., 8 define the following strategies:

(A5) 
$$\sigma_a(h_t) = (\hat{T}_t^s, 0) \text{ for } a = 9;$$

(A6) 
$$\sigma_a(h_t) = \begin{cases} (\hat{T}_t^s, \hat{E}_t) & \text{if } T_k^s = \hat{T}_k^s \text{ and } E_k = \hat{E}_k \text{ for all } k < t \text{ or } t = 1\\ (\hat{T}_t^s, 0) & \text{otherwise} \end{cases} \text{ for } a = 8;$$

(A7) 
$$\sigma_a(h_t) = (0, 0)$$
 for age  $a = 3, ..., \bar{a}_t(T^s) - 1$ ; and

(A8)

$$\sigma_a(h_t) = \begin{cases} (\hat{T}_t^s, \hat{E}_t) & \text{if } T_k^s = \hat{T}_k^s \text{ and } T_k^f = \hat{T}_k^f \text{ for all } k < t \text{ or } t = 1\\ (0, 0) & \text{otherwise} \end{cases} \text{ for } a = \bar{a}_t(T^s), \dots, 7.$$

A repetition of the arguments in step 1 shows that these strategies are an equilibrium and that they generate positive provision of social security and FIGs along the equilibrium path.

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